A NETWORK REDUCTION TECHNIQUE FOR MICROSTRIP THREE-DIMENSIONAL PROBLEMS

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ABSTRACT

A method is presented for obtaining the equivalent capacitances of microstrip discontinuity structures. The method of formulation and solution is based on the resistive network reduction technique. The formulation results in a matrix eigenvalue problem and permits us to obtain more accurate results with a relatively short computer time requirement. Numerical results for a variety of discontinuity configurations are presented and compared with results published by other investigators. Excellent agreement has been obtained in all cases.

Introduction

The problem of finding equivalent capacitances of microstrip discontinuities is of interest from a practical point of view in connection with the design and analysis of microwave integrated circuits and components. The purpose of this paper is to develop a straightforward and numerically effective method for determining the equivalent capacitances for a wide variety of microstrip discontinuity structures.

There are several methods that may be used to obtain the equivalent capacitances of microstrip discontinuity structures[1-6]. This paper describes a method employing the network reduction technique[7-9] that allows the determination of equivalent circuit parameters of partially shielded microstrip discontinuity structures. The formulation described in this paper is an extension of this network reduction technique which is applicable to microstrip discontinuity problems that are essentially three- rather than two-dimensional. The method involves dividing a microstrip structure under analysis into a number of equal subregions. A conductance matrix of the microstrip structure can be obtained by solving a three-dimensional resistive network in the finite difference form at each discrete node in the subregions. The main advantage of using the network reduction technique is that the conductance matrix is solved only at a finite number of discrete points at the interface of two dielectric media in the three-dimensional microstrip structure. This formulation results in a matrix eigenvalue problem and is regarded as a generalization for dealing with a wide variety of microstrip discontinuity structures. This technique compares favorably both in computation speed and accuracy with other numerical techniques published in the literature[1-6]. A description of this technique, along with its theoretical basis, is presented first, and then numerical results are presented for several specific discontinuity structures and compared with those results obtained in other methods by various earlier authors.

Basic Equations and Principle of Analysis

Basic Recurrence Relations and Conductance Matrices

Calculation of the conductance matrix of microstrip discontinuity structures is essentially reduced to the solution of a three-dimensional resistive network for regions determined by conducting strips in the structure. To illustrate the formulation of the general discontinuity problem, let us consider a finite section of microstrip line as shown in Figure 1a. It consists of a microstrip rectangular section of length L and width W. The spacing H between the conductor and the ground plane is assumed to be much less than the wavelength. The circuit is assumed to be lossless.

Figure 1b shows the equivalent terminal model corresponding to the structure in Figure 1a. The conductance matrix G at the dielectric interface is given as

$$G = G_u + G_\ell \qquad (1)$$

where G_u and G_ℓ are the conductance matrices of the upper and lower parts of the model, respectively. The method for calculation of the conductance matrices for two-dimensional cases has been given in an earlier paper[8].

Let us now demonstrate the application of the network reduction technique to the three-dimensional structure by developing an equivalent resistive network for the finite section of microstrip line shown in Figure 1c. The microstrip configuration under analysis is divided into a number of subregions, each intersect point is associated with six nodes. All conductances in a dielectric region are equal to the relative dielectric constant corresponding to the dielectric material. The conductances at the interface of two dielectric regions are equal to $(\varepsilon_{ri}+\varepsilon_{rj})/2$ where ε_{ri} and ε_{rj} are the relative dielectric constants of two adjacent materials, respectively. The manner in which resistances, conductances, and coordinates are defined is illustrated in Figure 1c. The conductance matrix G_k of the network at the kth level formed on the nodal analysis basis may be arranged to read

$$G_k = \left[G_{k-1}^{-1} + r_k U\right]^{-1} + g_k P \qquad (2)$$

where

G_{k-1} = MNXMN conductance matrix at the (k-1)th level of the resistive network

U = MNXMN identity matrix

$$P = \begin{bmatrix} J & -Q & & & O \\ -Q & J & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & -Q \\ O & & & -Q & J \end{bmatrix} = \text{MNXMN triple-diagonal matrix}$$

$$J = \begin{bmatrix} 4 & -1 & & & O \\ -1 & 4 & \ddots & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & -1 \\ O & & & -1 & 4 \end{bmatrix} = \text{MXM triple-diagonal matrix}$$

Q = MXM identity matrix

MN = M·N

This recurrence relation is derived by expansion in terms of the node equations of the three-dimensional resistive network. The admittance matrix may be seen to correspond with the result of two-dimensional cases given in [7]. The only difference is that, in the present three-dimensional case, the matrix P has a different form. In the matrix P all entries are zero except those on the diagonal and the two adjacent diagonals, and therefore explicit solutions for the admittance matrix are expected for the resistive network. One advantage in finding a solution in explicit form is

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that a considerable reduction in computing time can be gained. In addition, a considerable simplification is also gained in a double-subscript notation which is convenient for the computer.

Solution of the Triple-Diagonal Matrix

By applying the method for solving the characteristic value problems[10], the matrix formulation for obtaining an explicit expression for the eigenvalues and the corresponding normalized eigenfunctions of the matrix P has been derived. Details of the derivation of the solution are rather lengthy and will not be given here. The triple-diagonal matrix P has the eigenvalues

$$\lambda_k = 4\left[\sin^2\left(\frac{i\pi}{2(M+1)}\right) + \sin^2\left(\frac{j\pi}{2(N+1)}\right)\right] \tag{3}$$

and the corresponding normalized eigenfunctions

$$[L]_{k\ell} = \frac{2}{\sqrt{(N+1)(M+1)}} \sin\frac{jp\pi}{(N+1)} \sin\frac{iq\pi}{(M+1)} \tag{4}$$

where

$k = i + (j-1)\,M$
$\ell = q + (p-1)\,M$
$i,q = 1, 2, \ldots, M$
$j,p = 1, 2, \ldots, N$

It is evident that the eigenvalues and eigenfunctions for the three-dimensional case can be easily obtained by using Eqs. (3) and (4). In addition, in can be seen that the matrix L is a real, non-singular, symmetric and orthogonal matrix, and that a significant improvement of computing time will be obtained by storing only half of the total eigenfunction elements.

Computational Method

It is now quite simple to evaluate the conductance matrix for the microstrip structure shown in Figure 1a. The conductance matrix in (1), obtained by using a similarity transformation, can be expressed as[8]

$$G = L\,(\Gamma_u + \Gamma_\ell)L \tag{5}$$

or its corresponding resistance matrix is

$$R = L\,(\Gamma_u + \Gamma_\ell)^{-1}L \tag{6}$$

where

$\Gamma_u = \mathrm{diag}\,([\Gamma_u]_{1,1}, [\Gamma_u]_{2,2}, \ldots, [\Gamma_u]_{MN,MN})$
= eigenvalue of the upper part of the network

$\Gamma_\ell = \mathrm{diag}\,([\Gamma_\ell]_{1,1}, [\Gamma_\ell]_{2,2}, \ldots, [\Gamma_\ell]_{MN,MN})$
= eigenvalue of the lower part of the network

$$[\Gamma_u]_{\mu,\mu} = \sqrt{\lambda_\mu + \left(\frac{\lambda_\mu}{2}\right)^2} \qquad \mu = 1, 2, \ldots, MN$$

$$[\Gamma_\ell]_{\mu,\mu} = \lambda_\mu g_\ell + \cfrac{1}{r_\ell + \cfrac{1}{[\Gamma_{\ell-1}]_{\mu,\mu}}}$$

Substituting the eigenfunctions from Eq. (4), the entries of the conductance matrix can expand in the form

$$[G]_{k\ell} = \sum_{\zeta=1}^{N}\sum_{n=1}^{M} \frac{[\Gamma_u]_{\mu,\mu}+[\Gamma_\ell]_{\mu,\mu}}{(N+1)\;(M+1)}\left[\cos\frac{\pi(i-q)n}{M+1} - \cos\frac{\pi(i+q)n}{M+1}\right]\left[\cos\frac{\pi(j-p)\zeta}{N+1} - \cos\frac{\pi(j+p)\zeta}{N+1}\right] \tag{7}$$

where $\mu = n + (\zeta-1)\,M = 1, 2, \ldots, MN$

We have now succeeded in evaluating the resistance or conductance matrix at the interface of the two dielectric media in terms of the physical geometry. The next step is to compute a reduced terminal conductance or resistance matrix of the microstrip section by using the entire terminal conductance matrix derived above. This can be done by connecting the terminals on the microstrip section to a voltage source with all other terminals open-circuited. The reduced resistance matrix of the microstrip section is obtained by inverting the entire conductance matrix. Matrix elements corresponding to terminals which are open-circuited are excluded. Inversion of the reduced resistance matrix then yields the reduced conductance matrix or the equivalent charge profile existing in the microstrip section. Normalized capacitance associated with the microstrip section region can be calculated by multiplying the relative dielectric constant of the dielectric medium by the summation of all entries in the reduced conductance matrix.

We have so far dealt only with a finite section of a microstrip line. However, the concept presented here can be easily extended to other discontinuity structures such as microstrip gaps or an open-circuited microstrip line. The computed results for such discontinuity structures are given in the following section.

Numerical Results for Various Discontinuity Structures

Although some results have already been published for various discontinuity structures, it is interesting and instructive to compare results obtained from the present technique with those results obtained in other ways given in [1-3].

Capacitances of Microstrip Rectangular Sections

Figure 2 shows the numerical data obtained based on the network reduction technique for the capacitance of rectangular sections of microstrip and compared with published results by Farrar and Adams[1]. It can be seen from this figure that agreement between the two curves for three different dielectric constants is very good in a wide range of ratios H/W.

Open-Circuited Microstrip Lines

The effect of the discontinuity of an open-circuited microstrip line has been investigated by the present method. The results obtained for various W/H substrates are compared with those of Farrar and Adams[1] and Rahmat-Samii, Itoh and Mittra[2] in Figure 3. It can be seen that the results show very good agreement with those obtained by Farrar and Adams, and are about 7 percent higher than those obtained by Rahmat-Samii, et al, for a range of W/H of 0.3 to 2. One may observe that in Reference [2] the inclusion of higher order terms in the numerical computation will bring the results into better agreement with the results derived in the present paper.

Gap in Microstrip Lines

Figure 4 shows a microstrip of width W containing a gap of width S and the equivalent circuit for the gap discontinuity. Numerical results for the equivalent capacitances C_g and C_p for various gap widths with H = W = 0.508 mm and ε_r = 8.875 are shown in Figure 4 along with those given in [2,3]. Comparison shows that the results derived here agree well with those given by Maeda[3] for both the experimental and computed data.

Summary and Conclusions

In this paper a network reduction technique for solving three-dimensional microstrip problems is presented by the formulation of the matrix eigenvalue problem. These three-dimensional formulas presented may be regarded as generalizations of the two-dimensional results given in [7,8]. The explicit solution for finding characteristic values of the triple-diagonal matrix is derived in a form which has the advantage of a considerable reduction in computing time. Numerical results for several microstrip discontinuity geometrics are presented, and these results are compared with other computed data published in the literature confirming the accuracy of the calculations.

References

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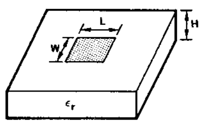

Figure 1a. Microstrip Rectangular Section

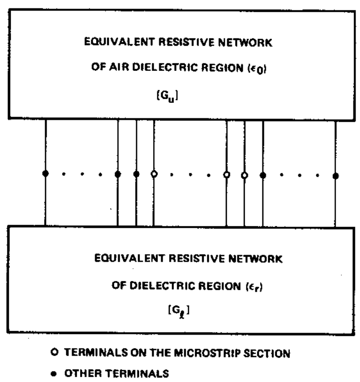

Figure 1b. Equivalent Terminal Model of the Microstrip Section

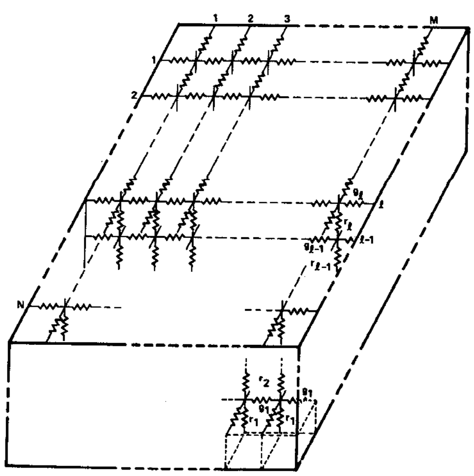

Figure 1c. Three-Dimensional Equivalent Resistive Network

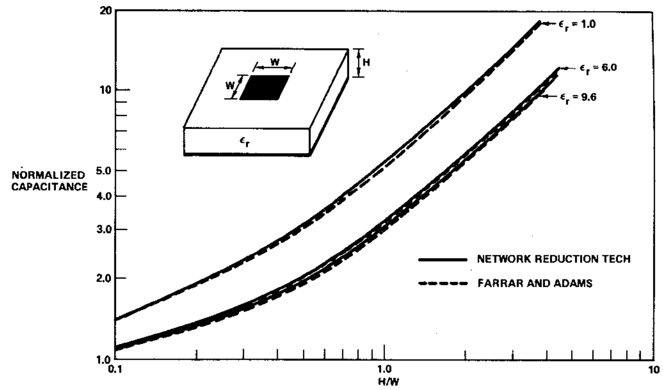

Figure 2. Capacitances of Microstrip Section for Various Dielectric Constants

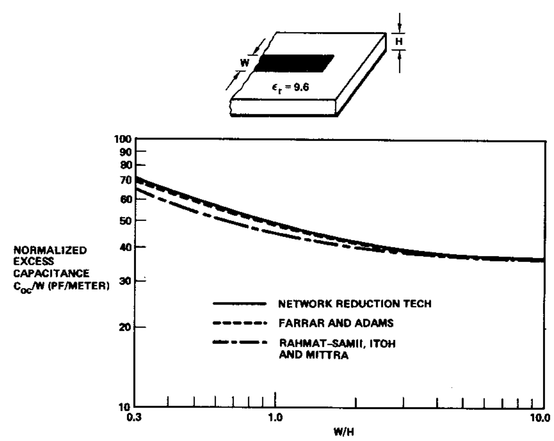

Figure 3. End Capacitance of an Open-Circuited Microstrip Line as a Function of Width-to-Height Ratio

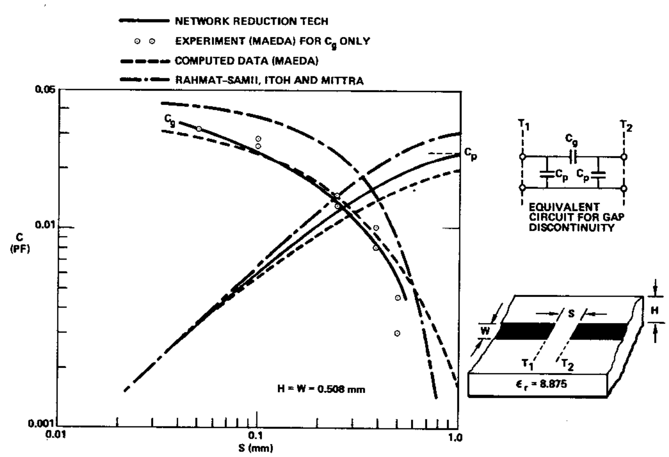

Figure 4. Gap Capacitances of a Microstrip Line for Various Gap Widths